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AN APPROXIMATE SOLUTION OF THE GENERALIZED STEFAN'S PROBLEM IN A POROUS MEDIUM

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Abstract—A generalized Stefan's problem for the coupled heat and mass transfer in a porous medium has been formulated, the solution of which provides the position of the variable evaporation front as well as the distribution of temperature and moisture in the porous body. It is shown that the effect of the deepening of the evaporation front on unsteady heat and mass transfer in a porous medium is characterized by ν, the nondimensional heat of vaporization. To bring out the effect of mass transfer on heat transfer with evaporation of liquid from porous system, results are compared with pure heat conduction in a half space. In two limiting cases above problem reduces to simple linear problems. It has been shown that in these cases the solutions obtained above lead to the corresponding solutions known earlier. The solution of this problem has also been obtained by an integral technique for comparison of the results obtained through the extended variational method based on local potential.

NOMENCLATURE

a_q, thermal diffusivity;
a_m, moisture diffusivity;
c_m, specific mass capacity;
c_q, specific heat capcity;
Fo, Fourier number, $\frac{a_q \tau}{l^2}$;
k, thermal conductivity;
Ko, Kossovitch number, $\frac{L c_m \Delta\theta}{c_q \Delta t_s}$;
l, the characteristic length;
L, latent heat of vaporization of liquid per unit mass;
Lu, Luikov number, $\frac{a_m}{a_q}$;
p, $\frac{k_{21}(t_v - t_0)}{(t_s - t_v)}$;
q, nondimensional thermal penetration depth;
Q, nondimensional moisture penetration depth;
$s(\tau)$, position of evaporation front;
$S(Fo)$, nondimensional position of evaporation front, s/l;
t, temperature;
t_s, temperature at surface $x = 0$;
T, nondimensional temperature, $\frac{t - t_0}{t_s - t_0}$;
x, length coordinate;
X, nondimensional length, x/l.

Greek symbols
α, nondimensional constant defined by (3.24);
β, nondimensional constant defined by (3.23);
δ, Soret coefficient;
$\Delta\theta$, $\theta_0 - \theta_v$;
Δt_v, $t_s - t_v$;
Δt_s, $t_s - t_0$;
ϵ, coefficient of internal evaporation;
θ, mass transfer potential;
Θ, nondimensional mass transfer potential, $\frac{\theta_0 - \theta}{\theta_0 - \theta_v}$;
λ, nondimensional constant defined by (3.1);
μ, $\frac{\varepsilon ko}{T_v}$;
ν, nondimensional latent heat of vaporization of liquid, $\frac{(1 - \epsilon)\rho_{mq} L}{c_q \Delta t_v}$;
ν_s, $\frac{(1 - \epsilon)\rho_m L a_q}{k_1(t_s - t_0)}$;
ρ_m, density of moisture per unit volume;
ρ_q, density of porous medium per unit volume;
ρ_{mq}, nondimensional density of liquid, ρ_m/ρ_q;
τ, time.

Subscripts
v, vaporizing state;
1, first region, $0 < x < s$;
2, second region, $s < x < \infty$;
21, ratio of properties of region 2 to 1;
0, initial state;

Superscript
0, macroscopic state.

1. INTRODUCTION

AN IMPORTANT group of problems is that in which a substance has a transformation point at which it changes from one phase to another with emission or absorption of heat. Such cases arise in many contexts and may involve melting, solidification or evaporation. First of all, the problem of thickness of polar ice was studied by Stefan and for this reason the problem of freezing is referred to as the problem of Stefan [1]. The essential new feature of such problems is the existence of a moving surface of separation between the two phases.

Luikov [2] has studied steady state heat and mass transfer between a capillary porous medium and an external gas stream during drying. It has been concluded by him that even with constant evaporation rate, evaporation takes place inside the body at a certain depth and that the deepening of the evaporation front has a very appreciable effect on the heat transfer rate. The above investigation was carried out with the assumption that depth of the evaporation front is fixed. It has been pointed out by Luikov [2] that in the case of intense drying and evaporation cooling of a porous body the evaporation front deepens into the body and thus the evaporation front is at a certain variable depth. The same problem was studied by Morgan and Yerazunis [3] for the laminar and turbulent gas streams with variable porous surface temperature. However, it may be appreciated that for the study of the initial stages of the drying process, a correct formulation should involve the solution of the transient problem of heat and mass transfer in a porous medium with variable front of evaporation and the position of the evaporation front should come out from the solution of the problem. To the best of our knowledge, no earlier attempt has been made to solve unsteady heat and mass transfer in a porous medium with moving evaporation front.

The moving evaporation front divides the system into two regions. While the moisture in one region is in vapour form only, in the other region it is in mixed (vapour and liquid) form. The moisture in vapour form can be assumed to be going out from the surface without taking any appreciable amount of heat from the system. Thus the problem reduces to the simultaneous solution of pure heat conduction problem in one region and solution of an unsteady coupled problem of heat and mass transfer with moving boundary in the other region. The essential new feature of this problem is the existence of a moving surface of separation between two regions where in one of the regions simultaneous transfer of heat and moisture takes place. This unsteady state problem characterized by a moving evaporation front in a porous medium and simultaneous transfer of heat and moisture may be called a generalized Stefan's problem.

The purpose of this paper is to formulate the generalized Stefan's problem in a porous medium, to obtain some approximate solutions of the same and to study the effect of deepening of evaporation front on temperature and moisture distributions. The solution of the problem of heat and mass transfer in a porous medium with moving evaporation front in conjunction with the boundary layer of the drying gas at the porous surface is still more formidable and would form the subject matter of another paper.

It is well known, see for example [1] that the problem of freezing is nonlinear and do not admit superposed solutions. Not many exact solutions have been obtained for such problems due to the inherent nonlinearity present in the system of equations describing the process. It is, therefore, usual to resort to approximate methods for obtaining solutions to the problem.

Kumar [4] has applied local potential method to the solution of nonlinear problems in heat and mass transfer in a porous medium. Rozenshtok [5] and Kumar and Narang [6] have applied heat balance integral techniques to obtain approximate solutions to some linear and nonlinear problems in heat and mass transfer in a porous medium. In this paper the solution of the problem is obtained by a boundary layer approach in local potential [7, 8]. Solution of this problem is derived by the integral technique [6] also, to make a comparison between the two results.

Numerical results for various values of nondimensional heat of vaporization v have been depicted graphically in Figs. 1–5. To bring out the effect of mass transfer on heat transfer with evaporation of liquid from porous system results are compared with pure heat conduction through a half space. The nondimensional rate of motion of evaporation front has been plotted against $\log_{10}(Fo)$ for various values of v. Further the effect of deepening of evaporation front on temperature and moisture distributions is shown in Figs. 2–5. In section 4 of this paper solutions of two limiting cases of the problem are derived from the solution obtained here and it is noted that these solutions are identical to the solutions of these problems known earlier. From numerical calculations it has been verified that the difference in the results obtained by heat balance integral technique [6] to those obtained by local potential is less than 5 per cent.

2. STATEMENT AND MATHEMATICAL FORMULATION OF THE PROBLEM

Consider the flow of heat and moisture through a porous half space ($x > 0$) in which the surface $x = 0$ is at temperature t_s, where t_s is greater than the vaporizing temperature of the liquid. It is well known that at a fixed pressure, there exists of every liquid a temperature at which it vaporizes completely. Let us assume that moisture vaporizes completely at temperature t_v. The surface at which moisture vaporizes completely is called the evaporation front. Initially, whole body is at temperature t_0 and moisture θ_0. Let the position of the evaporation front at time τ be given by $x = s(\tau)$. The evaporation front divides the porous system into two regions, in the region $0 < x < s$, moisture is in vapour form only and there is no moisture gradient, while in the region $s < x < \infty$ moisture is in mixed (vapour and liquid) form. Let us further assume that moisture potential at the evaporation front has a constant value θ_v. Moisture in vapour form can be assumed to be going out from the surface $x = 0$ without taking any appreciable amount of heat from the system.

Thus the problem reduces to the simultaneous solution of pure heat conduction problem in one region, and solution of a coupled problem of heat and mass transfer with moving boundary in the other region. The problem can be stated mathematically as follows:

$$\left.\begin{aligned} \frac{\partial t_1}{\partial \tau} &= a_q \frac{\partial^2 t_1}{\partial x^2} \quad &(2.1)\\ \theta_1 &= \theta_v \quad &(2.2)\end{aligned}\right] \quad 0 < x < s, \tau > 0.$$

The coupled equations of heat and mass transfer in the porous half space with constant properties can be stated as [9]

$$\left.\begin{aligned} \frac{\partial t_2}{\partial \tau} &= a_q \frac{\partial^2 t_2}{\partial x^2} + \frac{\epsilon L c_m}{c_q}\frac{\partial \theta_2}{\partial t} \quad &(2.3)\\ \frac{\partial \theta_2}{\partial \tau} &= a_m \frac{\partial^2 \theta_2}{\partial x^2} + a_m \delta \frac{\partial^2 t_2}{\partial x^2} \quad &(2.4)\end{aligned}\right] \quad s < x < \infty, \tau > 0.$$

As δ is small quantity, for many practical problems the second term in the R.H.S. of (2.4) may be neglected. The initial and boundary conditions can be stated as

$$t(0, \tau) = t_s \qquad \tau > 0 \qquad (2.5)$$

$$t(x, 0) = t_0 \qquad x > 0 \qquad (2.6)$$

$$\theta(x,0) = \theta_0 \qquad x > 0 \qquad (2.7)$$

$$\left.\begin{aligned} t_1 &= t_2 = t_v \quad &(2.8)\\ \theta_1 &= \theta_2 = \theta_v \quad &(2.9)\end{aligned}\right] \quad x = s.$$

An interface condition concerns the heat flux required in vaporizing the moisture at this evaporation front. As the evaporation front moves a distance ds, a quantity of heat per unit area is required for vaporizing the moisture at this surface. This requires

$$k_1 \frac{\partial t_1}{\partial x} - k_2 \frac{\partial t_2}{\partial x} = -(1 - \epsilon)\rho_m L \frac{ds}{d\tau} \quad x = s. \qquad (2.10)$$

The set of equations (2.1)–(2.10) can be represented in the nondimensional form as

$$\left.\begin{aligned} \frac{\partial T_1}{\partial Fo} &= \frac{\partial^2 T_1}{\partial X^2} \quad &(2.11)\\ \Theta_1 &= \Theta_v \quad &(2.12)\end{aligned}\right] \quad 0 < X < S$$

$$\left.\begin{aligned} \frac{\partial T_2}{\partial Fo} &= \frac{\partial^2 T_2}{\partial X^2} - \epsilon ko \frac{\partial \Theta_2}{\partial Fo} \quad &(2.13)\\ \frac{\partial \Theta_2}{\partial Fo} &= Lu \frac{\partial^2 \Theta_2}{\partial X^2} \quad &(2.14)\end{aligned}\right] \quad S < X < \infty.$$

The interface conditions are

$$\left.\begin{aligned} \frac{\partial T_1}{\partial X} - k_{21}\frac{\partial T_2}{\partial X} &= -v_s \frac{\partial S}{\partial Fo} \quad &(2.15)\\ T_1 = T_2 &= T_v \quad &(2.16)\\ \Theta_1 = \Theta_2 &= \Theta_v = 1 \quad &(2.17)\end{aligned}\right] \quad X = S.$$

The initial and boundary conditions are

$$T(X, 0) = 0 \qquad X > 0 \qquad (2.18)$$

$$\Theta(X, 0) = 0 \qquad X > 0 \qquad (2.19)$$

$$T(0, Fo) = 1 \qquad Fo > 0. \qquad (2.20)$$

It is well known, see for example [1] that Stefan's problem is nonlinear because of the moving front. Similarly, it can be seen that generalized Stefan's problem is nonlinear because of the boundary condition (2.15) at $X = S$. Hence solution of the problem in two regions are to be determined and cannot be superposed.

3. SOLUTION OF THE PROBLEM

In this section an approximate solution of the problem is obtained by local potential method [4, 7, 10]. The solution of the problem is also obtained by heat balance integral technique [6] for comparison of the results obtained by these two approaches. As explained in Section II of this paper the problem reduces to the solution of a pure heat conduction problem in one region, and a coupled problem of heat and mass transfer in the other region.

(a) *Solution for the region* $0 < x < S$

For the temperature distribution in the region $0 < X < S$ the exact solution for a semi-infinite body is used. This approximation has also been used by Tien and Geiger [11] and Cho and Sunderland [12] under the assumption that the law of motion of surface of separation is given by

$$S(Fo) = 2\lambda\sqrt{Fo} \tag{3.1}$$

where λ is to be determined later.

Thus

$$T = \frac{(T_v - 1)}{\operatorname{erf}(\lambda)}\operatorname{erf}\left(\frac{X}{2\sqrt{Fo}}\right) + 1 \quad 0 < X < S. \tag{3.2}$$

Since there is no moisture gradient in this region, therefore, moisture is always Θ_v and in vapour form

$$\Theta_1 = \Theta_v. \tag{3.3}$$

(b) *Solution for the region* $S < X < \infty$

In the region $S < X < \infty$, a coupled problem of heat and mass transfer with first kind of boundary conditions is to be solved. The solution of this problem is obtained by a boundary layer approach in local potential [7, 8]. Schechter [8] and Kumar and Gupta [7] have used idea of penetration depth in local potential. Let us define a non-dimensional thermal penetration distance $q(Fo)$ beyond which it is assumed that no heat flow takes place, and a non-dimensional mass penetration distance $Q(Fo)$ beyond which no mass transfer takes place. These conditions can be mathematically written as

$$T_2 = 0 \quad \text{at} \quad X = S + q \tag{3.4}$$

$$\frac{\partial T_2}{\partial X} = 0 \quad \text{at} \quad X = S + q \tag{3.5}$$

and

$$\Theta_2 = 0 \quad \text{at} \quad X = S + Q \tag{3.6}$$

$$\frac{\partial \Theta_2}{\partial X} = 0 \quad \text{at} \quad X = S + Q. \tag{3.7}$$

Now with the help of conditions (2.16), (2.17) and (3.4)–(3.7) we can assume parabolic profiles for the temperature and moisture distributions as

$$T_2 = T_v\left(1 - \frac{X - S}{q}\right)^2 \tag{3.8}$$

$$\Theta_2 = \left(1 - \frac{X - S}{Q}\right)^2. \tag{3.9}$$

Kumar [4] has established the form of local potential for coupled problem of heat and mass transfer in a porous medium. For one dimensional case it can be written as

$$J = \int\left[\frac{1}{2}\left(\frac{\partial T_2}{\partial X}\right)^2 + T_2\frac{\partial T_2^0}{\partial F_0} + \epsilon Ko T_2\frac{\partial \Theta_2^0}{\partial Fo} + \frac{Lu}{2}\left(\frac{\partial \Theta_2}{\partial X}\right)^2 + \Theta\frac{\partial \Theta_2^0}{\partial Fo}\right]\mathrm{d}X. \tag{3.10}$$

Here, variation is to be taken independently and exclusively over T and Θ, where

$$T = T^0 + \delta T \tag{3.11}$$

$$\Theta = \Theta^0 + \delta\Theta. \tag{3.12}$$

The unknown parameters q and Q in the profiles (3.8) and (3.9) are to be determined from

$$\frac{\partial J}{\partial q} = 0 \tag{3.13}$$

and

$$\frac{\partial J}{\partial Q} = 0 \tag{3.14}$$

with the self consistency conditions [10]

$$q = q^0 \tag{3.15}$$

and

$$Q = Q^0 \tag{3.16}$$

Rozenshtok [5] and Kumar and Narang [6] have used the idea of penetration depth for the solution of coupled problems of heat and mass transfer. In the case of coupled phenomena of heat and mass transfer, it is essential to differentiate the cases where heat transfer precedes mass transfer in the initial stage of the process or vice versa. Therefore, two different cases are to be considered, firstly when heat transfer precedes mass transfer ($q(Fo) > Q(Fo)$) and secondly when mass transfer precedes heat transfer ($Q(Fo) > q(Fo)$).

Case 1. When $q > Q$ (i.e. when heat transfer precedes mass transfer).

In this case $S + q > S + Q$, therefore, the limits of integration in (3.10) will be from S to $S + q$. The difference in solution in the two cases arise due to the presence of the term

$T_2\frac{\partial \Theta_2^0}{\partial Fo}\mathrm{d}X$ in (3.10). Here

$$\int_S^{S+q} T_2\frac{\partial \Theta_2^0}{\partial Fo}\mathrm{d}X = \int_S^{S+Q} T_2\frac{\partial \Theta_2^0}{\partial Fo}\mathrm{d}X + \int_{S+Q}^{S+q} T_2\frac{\partial \Theta_2^0}{\partial Fo}\mathrm{d}X. \tag{3.17}$$

Since

$$\frac{\partial \Theta_2^0}{\partial Fo} = 0 \text{ when } S + Q < X < S + q. \quad (3.18)$$

Therefore, (3.17) takes the form

$$\int_S^{S+q} T_2 \frac{\partial \Theta_2^0}{\partial Fo} \mathrm{d}X = \int_S^{S+Q} T_2 \frac{\partial \Theta_2^0}{\partial Fo} \mathrm{d}X. \quad (3.19)$$

On simplification the set of equations (2.15), (3.13) and (3.14) may be written as

$$2Q\dot{Q} + 5\dot{S}Q = 10Lu \quad (3.20)$$

$$(2q\dot{q} + 5\dot{S}q - 10) + 15\mu\left[\dot{S}Q\left(\frac{1}{3} - \frac{1}{6}\frac{Q}{q}\right) + 2\dot{Q}\left(\tfrac{1}{6}Q - \frac{Q^2}{10q}\right)\right] = 0 \quad (3.21)$$

$$\frac{e^{-\lambda^2}}{\sqrt{(Fo)} \times \sqrt{(\pi)}\,\mathrm{erf}(\lambda)} - \frac{2k_{21}(t_v - t_0)}{(t_s - t_v)\,q} = v\dot{S}. \quad (3.22)$$

Where dot (.) denotes differential w.r.t. Fo.

Now the problem reduces to obtaining $S(Fo)$ and parameters q and Q from the set of equations (3.20)–(3.22). Goodman [13], Biot [14] and Kumar and Narang [6] have shown that penetration distance varies as $A\sqrt{Fo}$ where A is some constant to be determined from the equations. Therefore we assume

$$q = 2\beta\sqrt{Fo} \quad (3.23)$$

$$Q = 2\alpha\sqrt{Fo}. \quad (3.24)$$

Now the set of equations (3.20)–(3.22) can be written as

$$4\alpha^2 + 10\alpha\lambda = 10Lu \quad (3.25)$$

$$(4\beta^2 + 10\beta\lambda - 10) + 15\mu[2\alpha\lambda(\tfrac{1}{3} - \tfrac{1}{6}\alpha/\beta) + 2\alpha(\tfrac{1}{3}\alpha - \tfrac{1}{5}\alpha^2/\beta)] = 0 \quad (3.26)$$

$$\frac{e^{-\lambda^2}}{\sqrt{(\pi)}\,\mathrm{erf}(\lambda)} - \frac{p}{\beta} = v\lambda. \quad (3.27)$$

The set of parameters α, β, λ can be obtained from set of equations (3.25)–(3.27). Further the rate of motion of evaporation front is given by

$$\dot{S} = \frac{\lambda}{\sqrt{Fo}}. \quad (3.28)$$

Rozenshtok [5] and Kumar and Narang [6] have applied integral techniques to obtain approximate solutions of coupled problem of heat and mass transfer. Proceeding in the same way as Kumar and Narang [6] and assuming profiles for temperature and moisture distributions as (3.8) and (3.9) the corresponding set of equations for determination of α, β and λ come out as

$$2\alpha^2 + 6\alpha\lambda = 6Lu \quad (3.29)$$

$$(2\beta^2 + 6\beta\lambda - 6)\alpha = -6\mu\beta \quad (3.30)$$

$$\frac{e^{-\lambda^2}}{\sqrt{(\pi)}\,\mathrm{erf}(\lambda)} - \frac{p}{\beta} = \lambda v. \quad (3.31)$$

Therefore, the set of equations (3.2), (3.8) and (3.9) give us the temperature and moisture distributions in the two regions with these determined values of α, β and λ when $q > Q$.

Case 2. When $q < Q$ (i.e. when mass transfer precedes heat transfer).

In this case $S + q < S + Q$, therefore, the limits of integration in (3.10) will be from S to $S + Q$. Here

$$\int_S^{S+Q} T_2 \frac{\partial \Theta_2^0}{\partial Fo} \mathrm{d}X = \int_S^{S+q} T_2 \frac{\partial \Theta_2^0}{\partial Fo} \mathrm{d}X + \int_{S+q}^{S+Q} T_2 \frac{\partial \Theta_2^0}{\partial Fo} \mathrm{d}X. \quad (3.32)$$

Since $T_2 = 0$ when $S + q < X < S + Q$. (3.33)

Therefore, (3.32) takes the form

$$\int_S^{S+Q} T_2 \frac{\partial \Theta_2^0}{\partial Fo} \mathrm{d}X = \int_S^{S+q} T_2 \frac{\partial \Theta_2^0}{\partial Fo} \mathrm{d}X. \quad (3.34)$$

The difference in relations (3.19) and (3.34) is in their limits of integration.

Proceeding in the same way as in Case 1, equations (3.25) and (3.27) remain same, only equation (3.26) changes, the changed equation may be written as

$$(4\beta^2 + 10\beta\lambda - 10)\alpha + 15\mu[2\beta^2\lambda(\tfrac{1}{3} - \tfrac{1}{6}\beta/\alpha) + 2\beta^2(\tfrac{1}{3}\beta - \tfrac{1}{5}\beta^2/\alpha)] = 0. \quad (3.35)$$

Thus in this case the set of parameters α, β and λ is to be obtained from set of equations (3.25), (3.27) and (3.35).

Further, proceeding by heat balance integral [6] it is seen that the set of equations (3.29) and (3.31) remain same as in Case 1 and only equation (3.30) changes, the changed equation may be written as

$$(2\beta^2 + 6\beta\lambda - 6)\,\alpha^2 = -6\mu\beta^2. \quad (3.36)$$

Now the set of parameters α, β and λ is to be obtained from set of equations (3.29), (3.31) and (3.36).

Therefore, the set of equations (3.2), (3.3), (3.8) and (3.9) with these determined values of α, β and λ give us the temperature and moisture distributions in the two regions when $q < Q$.

4. SOME PARTICULAR CASES OF THE PROBLEM

In this section we discuss the two limiting cases of the problem for $\lambda \to 0$ and $\lambda \to \infty$.

Case 1. $\lambda = 0$

When $\lambda = 0$, equation (3.1) implies that $S = 0$ for all values of Fo. It is possible only when the initial and boundary conditions are such that the evaporation front is fixed at the surface. From statement of the problem in Section 2 it is obvious that λ can be zero, only when the temperature at the surface is less than or equal to the vaporizing temperature of the liquid, therefore, complete vaporization cannot take place inside the porous system, consequently, the problem reduces to coupled heat and mass transfer problem in a porous half space with first kind of boundary conditions.

Taking $\lambda = 0$ in the set of equations (3.29) and (3.30) we get

$$2\alpha^2 = 6Lu \tag{4.1}$$

$$(2\beta^2 - 6)\,\alpha = -6\mu\beta. \tag{4.2}$$

Since this is a one phase problem, therefore, the equation (3.31) is redundant in this case. From equations (4.1) and (4.2) we obtain

$$Q = \sqrt{(12LuFo)} \tag{4.3}$$

$$q = [-\sqrt{(3/Lu)}\,\mu + \sqrt{(12 + 3\mu^2/Lu)}]\sqrt{Fo}. \tag{4.4}$$

The solution of the problem given by equations (4.3), (4.4), (3.8) and (3.9) is identical to the one obtained by Kumar and Narang [6] for the case when heat transfer precedes mass transfer.

Taking $\lambda = 0$ in the set of equations (3.29) and (3.36) we get

$$2\alpha^2 = 6Lu \tag{4.5}$$

$$(2\beta^2 - 6)\,\alpha^2 = -6\mu\beta^2. \tag{4.6}$$

From equations (4.5) and (4.6) we obtain

$$Q = \sqrt{(12LuFo)} \tag{4.7}$$

$$q = \sqrt{(12Lu/(Lu + \mu))Fo} \tag{4.8}$$

Now set of equations (4.7), (4.8), (3.8) and (3.9) represent the solution of the problem for the case when mass transfer precedes heat transfer ($Q > q$). This solution is identical to the one obtained by Kumar and Narang [6].

Case 2. $\lambda = \infty$

When $\lambda \to \infty$, equation (3.1) implies that $S \to \infty$ for all values of Fo. It is possible only when the initial and boundary conditions are such that either no moisture is present at all or the moisture is present in vapour form only. From statement of the problem in Section II it is obvious that λ can be infinity only when the initial temperature of the body is T_v or greater than T_v and the surface $X = 0$ is at a fixed temperature greater than the initial temperature. Thus the problem reduces to pure heat conduction through a half space with first kind of boundary conditions. The solution of this problem can be obtained from equation (3.2) by taking $\lambda = \infty$, the equation (3.2) reduces to

$$T = \mathrm{erfc}\left(\frac{X}{2\sqrt{Fo}}\right) \tag{4.9}$$

As quoted at Carslaw and Jaeger [1], (4.9) is the exact analytical solution of the problem.

Thus it is seen that in the limiting cases $\lambda \to 0$ and $\lambda \to \infty$ results obtained here are identical to the corresponding results known earlier.

5. RESULTS AND DISCUSSION

The results of numerical calculations for various values of ν are depicted in Figs. 1–5. In Fig. 1 nondimensional rate of motion of evaporation front $\dot{S}$ is

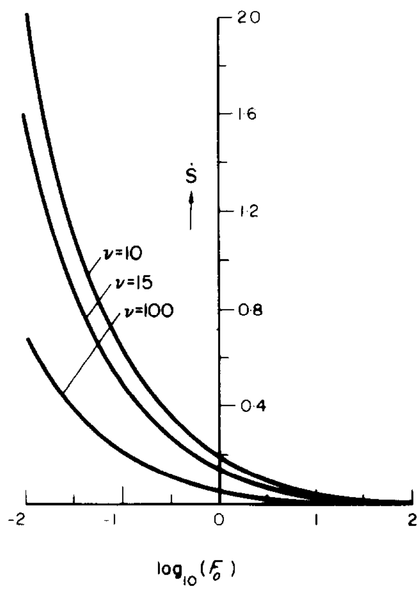

FIG. 1. Effect of variability of ν on $\dot{S}(\log_{10}(Fo))$ for $Lu = 0{\cdot}5$, $\varepsilon = 0{\cdot}5$, $Ko = 1{\cdot}2$, $k_{21} = 1{\cdot}0$ and $p = 1{\cdot}0$.

plotted against $\log_{10}(Fo)$ for three different values of nondimensional heat of vaporization $\nu = 10$, 15 and 100. It is seen that $\dot{S} \to 0$ as $\nu \to \infty$ and $\dot{S} \to \infty$ as $\nu \to 0$ for all values of Fo. Consequently the evaporation front becomes fixed as ν approaches infinity. Hence when ν is large evaporation front very close to the surface.

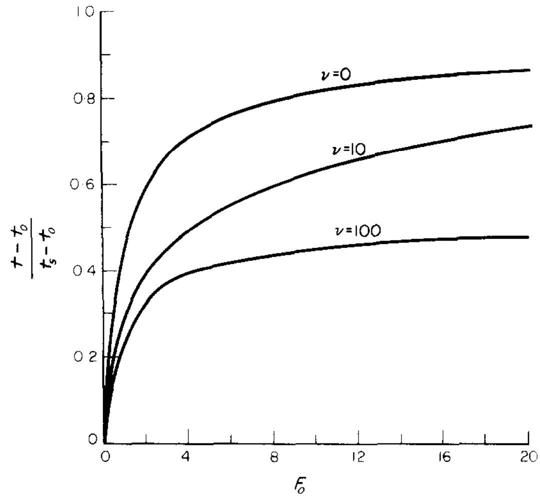

FIG. 2. Effect of variability of ν on $T(1, Fo)$ for $Lu = 0{\cdot}5$, $\varepsilon = 0{\cdot}5$, $Ko = 1{\cdot}2$, $k_{21} = 1{\cdot}0$ and $p = 1{\cdot}0$.

Since

$$\nu = \frac{(1-\epsilon)\,\rho_{mq}L}{c_q \Delta t_v}. \tag{5.1}$$

Therefore, for a given liquid ν increases as Δt_v decreases. Thus ν is large implies that the surface temperature is very near to the vaporizing temperature of the liquid. Hence there is not intense drying at the surface. In such cases Luikov [2] has reported that evaporation front is very close to the surface.

In Figs. 2 and 3 nondimensional temperature profile (T vs Fo for $X = 1{\cdot}0$) and (T vs X for $Fo = 1{\cdot}0$) have been depicted for three different values of $\nu = 0$, 10 and 100. Here $\nu = 0$ represents the case of pure heat conduction through a half space. Also from numerical calculations it is seen that the figures for $\nu = 100$ and $\nu = \infty$ representing the temperature distributions are almost identical. From Figs. 2 and 3 it is seen that the temperature at a fixed position decreases as parameter ν increases. Hence it can be concluded that the effect of mass transfer ($\nu > 0$) on heat transfer with evaporation of liquid from porous system results in the decrease of temperature. Thus for $\nu = 100$ the temperature at $X = 1{\cdot}0$ and $Fo = 1{\cdot}0$ is about 45 per cent lower than the case when no moisture is present. Since some amount of heat flux is consumed in evaporating the liquid, therefore, the above stated result is in accordance with the physical expectations.

From the observations in Figs. 1–3 it is noted that when the evaporation front deepens (e.g. $\nu = 10$) the temperature at a fixed position is higher than that with evaporation on the surface ($\nu = \infty$). This is an extension of the result obtained by Luikov [2] for a steady state problem.

In Figs. 4 and 5 nondimensional moisture potential (Θ vs Fo for $X = 1{\cdot}0$) and (Θ vs X for $Fo = 1{\cdot}0$) have been depicted for three different values of $\nu = 10$, 100 and ∞. Here $\nu = \infty$ represents the case of heat and mass transfer through a porous half space when the evaporation front is fixed at the surface. The nondimensional moisture potential decreases as the moisture potential increases and vice versa. It is seen

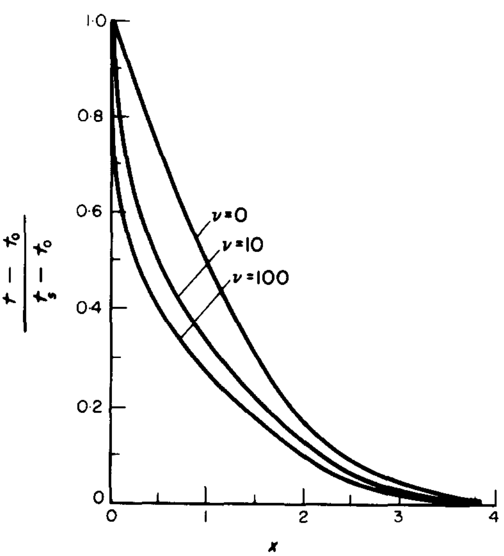

FIG. 3. Effect of variability of ν on $T(X, 1)$ for $Lu = 0{\cdot}5$, $\varepsilon = 0{\cdot}5$, $Ko = 1{\cdot}2$, $k_{21} = 1{\cdot}0$ and $p = 1{\cdot}0$.

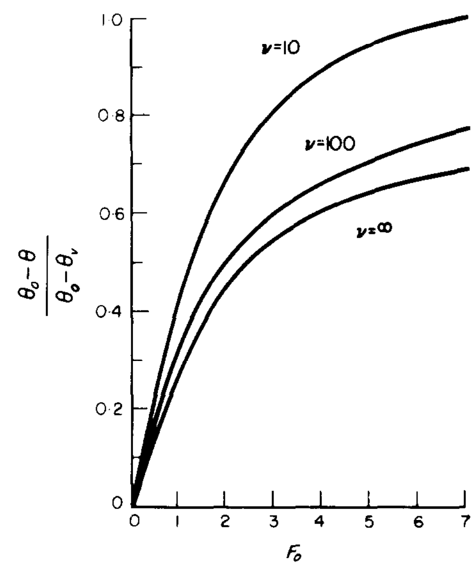

FIG. 4. Effect of variability of ν on $\Theta(1, Fo)$ for $Lu = 0{\cdot}5$, $\varepsilon = 0{\cdot}5$, $Ko = 1{\cdot}2$, $k_{21} = 1{\cdot}0$ and $p = 1{\cdot}0$.

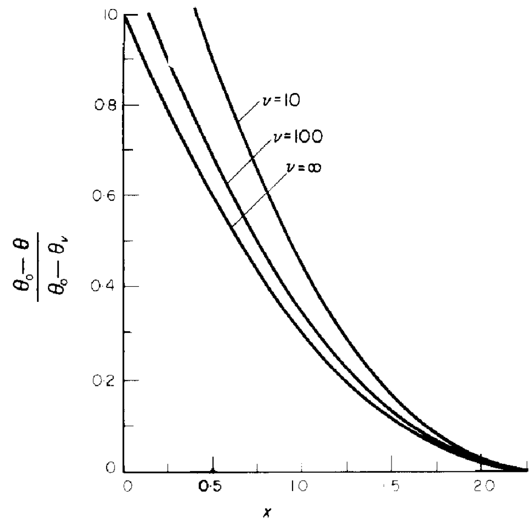

FIG. 5. Effect of variability of ν on $\Theta(X,1)$ for $Lu = 0{\cdot}5$, $\varepsilon = 0{\cdot}5$, $Ko = 1{\cdot}2$, $k_{21} = 1{\cdot}0$ and $p = 1{\cdot}0$.

from Figs. 4 and 5 that moisture distribution for a fixed position increases as parameter ν increases. Hence, it can be concluded that as rate of motion of evaporation front increases, the moisture potential decreases. Thus for $\nu = \infty$ the moisture potential at $X = 1{\cdot}0$ and $Fo = 1{\cdot}0$ is about 35 per cent higher than the case when $\nu = 10$.

From the observations in Figs. 1, 4 and 5 it is noted that as the evaporation front deepens (e.g. $\nu = 10$) the moisture potential is lower than that with evaporation on the surface ($\nu = \infty$).

The two limiting cases of the problem discussed in section 4 of this paper can be easily seen from Figs. 2–5. From numerical calculations it is seen that as $\nu = \infty$, $\lambda \to 0$ and as $\nu \to 0$, $\lambda \to \infty$.

Thus the solutions of the limiting cases of the problem are the solutions for the cases $\nu = 0$ and $\nu = 100$ represented in Figs. 2 and 3 and $\nu = \infty$ represented in Figs. 4 and 5. Further from numerical calculations it has been verified that the difference between the results obtained by integral technique [6] to those obtained by local potential method [4, 7] is less than 5 per cent and hence is not shown in figures.

In process of transpiration cooling, intense drying of porous system and in many other practical problems $0 < \nu < \infty$. From discussion in this section it is obvious that the rate of motion of evaporation front has a very appreciable effect on the temperature and moisture distributions inside the porous system. It may be concluded that the nondimensional heat of vaporization ν characterizes the effect of the deepening of the evaporation front on unsteady state heat and mass transfer in a porous system.

6. CONCLUSION

In this paper generalized Stefan's problem with moving evaporation front in a porous body has been formulated and its approximate solution has been obtained by local potential method. The solution of the problem is also obtained by an integral technique to make a comparison between the two results. It is concluded in the study that the temperature at a fixed position decreases and moisture at the fixed position increases as nondimensional heat of vaporization ν increases.

Further, it is shown that the nondimensional heat of vaporization ν characterizes the effect of deepening of the evaporation front on unsteady state heat and mass transfer in a porous medium. It is noted that the position and rate of motion of evaporation front are dependent on a parameter ν. Solutions of limiting cases of the problem discussed in section 4 of this paper are obtained from the solution of the generalized Stefan's problem by taking $\lambda = 0$ and ∞. Solutions obtained in these particular cases are identical to the solutions known earlier. Moreover, it is seen that when ν is large evaporation front is very close to the surface.

The study of this problem will find direct applications in various fields like transpiration cooling of Turbine blades, in chemical reactions when the reaction front is moving and in the re-entry problems.

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UNE SOLUTION APPROCHÉE DU PROBLÈME GÉNÉRALISÉ DE STEFAN DANS UN MILIEU POREUX

Résumé—On formule un problème généralisé de Stefan pour le transfert couplé de masse et de chaleur. La solution donne la position du front d'évaporation dans le corps poreux, aussi bien que la distribution de température et d'humidité. On montre que l'effet de déplacement du front d'évaporation sur le transfert transitoire de chaleur et de masse dans un milieu poreux est caractérisé par v, la chaleur de vaporisation adimensionnelle. Pour dégager l'effet du transfert massique sur le transfert thermique avec évaporation de liquide, les résultats sont comparés avec la conduction pure dans un demi-espace. Dans deux cas limites le problème se réduit à des problèmes linéaires simples. On montre que dans ces cas les solutions obtenues conduisent à des formes déjà connues. La solution de ce problème a aussi été trouvé par une technique intégrale, pour comparaison des résultats obtenus par une méthode variationnelle basée sur le potentiel local.

EINE NÄHERUNGSLÖSUNG DES ALLGEMEINEN STEFAN- PROBLEMS IN EINEM PORÖSEN MEDIUM

Zusammenfassung—Es wurde ein verallgemeinertes Stefan-Problem für den gleichzeitigen Wärme- und Stoffübergang in einem Medium formuliert. Die Lösung ergibt den Anteil der veränderlichen Verdampfungsfront und die Temperatur und Feuchteverteilung im porösen Körper. Es wird gezeigt, dass der Effekt des Absinkens der Verdampfungsfront bei instationärem Wärme- und Stoffaustausch in einem porösen Körper durch die dimensionslose Verdampfungswärme charakterisiert werden kann. Um die Auswirkungen des Massentransports auf die Wärmeübertragung bei Verdampfung von Flüssigkeit aus porösen Körpern zu zeigen, wurden die Ergebnisse mit denen der reinen Wärmeleitung im halbunendlichen Körper verglichen. In zwei Grenzfällen vereinfacht sich das Problem in ein einfaches lineares. Es wird gezeigt, dass die erhaltenen Lösungen zu entsprechenden bereits bekannten Ergebnissen führen. Die Lösung des Problems wurde auch nach einer Integral—Technik ermittelt im Vergleich zu Ergebnissen, die nach der erweiterten Variationsmethode auf Grund des lokalen Potentials erhalten wurden.

ПРИБЛИЖЕННОЕ РЕШЕНИЕ ОБОБЩЕННОЙ СТЕФАНОВСКОЙ ЗАДАЧИ В ПОРИСТОЙ СРЕДЕ

Аннотация—Сформулирована задача для совместного тепло- и массообмена в пористой среде. Ее решение позволяет определить положение фронта перемещенного испарения, а также распределение температуры и влажности в пористом теле. Показано, что влияние заглубления фронта испарения на нестационарный тепло- и массообмен в пористой среде характеризуется безразмерной теплотой испарения v. Для выяснения влияния переноса массы на теплообмен при испарении жидкости из пористой системы проведено сравнение результатов, полученных в случае чистой теплопроводности в полупространстве. В двух предельных случаях указанная задача сводится к простым линейным случаям. Показано, что в этих случаях решения, полученные выше, приводят к соответсвующим решениям, известным ранее. Решение рассматриваемой задачи проведено ракже интегральным методом, что позволяет сравнить результаты с данными, полученными обощенным вариационным методом, основанным на локальном потенциале.